

# **Foolish Formulas**

Breaking the SMR grip on NHS public health funding allocations

Mervyn Stone

April 2014

Civitas: Institute for the Study of Civil Society

**Mervyn Stone** is an emeritus professor of statistics at University College London where he has been teacher, researcher and statistical adviser since 1968—before that, at the universities of Cambridge, Princeton, Aberystwyth, Michigan and Durham. He is the author of Civitas book *Failing to Figure*.



55 Tufton Street, London SW1P 3QL  
T: 020 7799 6677  
E: [info@civitas.org.uk](mailto:info@civitas.org.uk)

Civitas: Institute for the Study of Civil Society is an independent think tank which seeks to facilitate informed public debate. We search for solutions to social and economic problems unconstrained by the short-term priorities of political parties or conventional wisdom. As an educational charity, we also offer supplementary schooling to help children reach their full potential and we provide teaching materials and speakers for schools.

Civitas is a registered charity (no. 1085494) and a company limited by guarantee, registered in England and Wales (no. 04023541).

---

1. 2002 was a good year—and Edinburgh a rich forum—for social policy-makers to influence government and vice versa. In one direction, the Information and Statistics Division of NHS Scotland received the influential AREA report<sup>1</sup> from an eight-strong research team (nominally Glaswegian but mostly health economists from England). In the other direction, the Scottish Executive commissioned a nine-strong (Oxfordian) team to construct *Indices of Deprivation for Scotland*<sup>2</sup>.

2. The AREA report was taken as justification for the foolish<sup>3</sup> formulas that England's Department of Health finance department have been using for over a decade to fund primary care trusts. Now, in its two formulas for allocating over 60 billions to clinical commissioning groups (CCGs), NHS England's finance department is changing the justification but not the foolishness.

The disposal of 90% is by a long, irrational and evidence-free formula that has striking financial features that are not understood by the health economists who fabricated the formula. (The hard evidence for that accusation lies in the demonstrably illogical defences of some questionable coefficients of the formula.<sup>4</sup>) But the concern of this note is with the second formula<sup>5, 6, 7</sup> for the remaining 10%—which is not so much a formula as a prescription for how to base its earmarked funding on small-area values of a single official statistic, the standard mortality rate SMR<75 for under-75s.

3. The grip of SMR<75 on the thinking of Department of Health (DH) policy-makers in the design of the local authority public health funding formula is documented<sup>8</sup> by a DH team accountable to the department's Advisory Committee on Resource Allocation. The shape of an interim version of the formula was respectfully disputed by some local authorities and other concerned bodies, but their arguments were analysed and dismissed in two more DH reports<sup>9, 10</sup>, followed by a ripple of uncertainty in a third<sup>11</sup>. The uncertainty ended with the requirement of NHS England's chief finance officer, that the formula for Clinical Commissioning Groups should be based on the SMR<75, *weighted in a similar way to the local authority public health grant formula*<sup>5</sup>.

4. Unfortunately, the requirement replaced the uncertainty about which formula was to be used with an ambiguity about its objective—already boldly embedded in the requirement that *NHS England should make a further adjustment [the 10%] for inequalities/unmet need when considering how to allocate funds*. The ambiguity

extends the local authority objective (the reduction of public health 'inequality' alone) by encompassing a supposed 'unmet need' for acute care provision by sections of the population—sections that do not make full use of available hospital services. (The term 'unmet need' had been trumpeted by Secretary of State Milburn when he launched the AREA-based formula in 2002 and was used by Secretary of State Hewitt when she defended the formula against the charge of unfairness.) The ambiguity matters! Without a reasonably clear objective for the 10%, how can anyone provide a rationale for differences between CCG allocations that come from the SMR-based component?

5. The standard mortality rate of any section of a population over some period is the ratio of the observed number of deaths in the period (as numerator) to the number of deaths (as denominator) that there would have been if the different age-bands in the section had had national death-rates. The 10% formula uses SMR<75 as the basis for weighting the populations of the roughly 7,000 Middle Layer Super Output Areas (MSOAs) that cover England much like electoral wards. DH's earlier adoption of SMR<75-based weighting of small area sub-populations of local authorities had not been given any introductory rationale—only the assertion:

*The SMR < 75 years is a major driver in the public health formula. It is applied at MSOA level in order to take account of inequality within local authorities as well as between local authorities.*<sup>10</sup>

6. An SMR is here an index for an asymmetric comparison of the age-band death-rates in a sub-population and those of the nation (the age-band death-rates of the sub-population are multiplied by age-band populations to give the total number of deaths as numerator). Exchange the roles of sub-population and nation and you get a different number. To put the emphasis back on the sub-population, you then have to exchange numerator and denominator i.e. take the reciprocal. The number you end up with is the Comparative Mortality Factor (CMF). The crucial distinction between SMR and CMF is that SMR is a function of both sub-population age-band death-rates and sub-population age-band populations, whereas for CMF you only need the death rates.

7. Why did the DH policy-makers favour statistics-hungry SMR over statistics-meagre CMF? When he worked in the Office of Population Census and Surveys on the scientific understanding of mortality before joining the Department of Health to become its Head of Profession, medical statistician John Fox's judgement<sup>12</sup> was that CMF is *the correct measure to use if the mortality rates for sub-populations are*

---

*to be compared among themselves as well as with those for the standard population—and that in cases where the age distribution of the population studied differs from either the standard population age distribution or the age distributions of other populations with which comparisons are made, the SMR may be misleading.*

8. John Fox's judgement that CMF is better than SMR gets apparently strong support in Appendix 3 of the Oxford study<sup>2</sup>, once you have replaced Scottish 'deprivation' by DH's 'inequality' and electoral wards by MSOAs. CMF was one of the Scottish indices contributing to a composite measure of deprivation, concerning which the study makes two claims:

*an ideal measure of area deprivation would be able to quantify deprivation at the level of 'neighbourhoods' or small areas with a homogeneity of characteristics and a standard population size<sup>2</sup>.*

which draws an approximate parallel with the DH work, and the ambitious claim that

*such a measure would be able to offer to policy makers and funding programmes a robust method of describing the geographic distribution of deprivation.*

which gives hostages to fortune. Perhaps wisely, the Oxford team left it at that, without considering either the rationale of using CMF as the basis of a funding programme for small areas or the sense in which its application could be described as robust. In this note, we are trying to address the questions here ignored, as applied to SMR-based funding.

9. Appendix 3 of *Scottish Indices of Deprivation*<sup>2</sup> found fault with SMR in a hypothetical but illuminating example, which is reminiscent (to statisticians) of the famous 'Simpson's paradox':

Two sub-populations (A and B); two age groups ('young' and 'old'); A is 60% young, B is 80% young; for A, death-rates are 10% and 25%, respectively; for B, death-rates are 9% and 24%; national death-rates are 5% and 22%. If allocation is to be based on CMF, then, because  $10\% > 9\%$  and  $25\% > 24\%$ , area A would get more than B, quite reasonably. But the SMR for A is the ratio of the Crude death-rate to Expected death-rate i.e.

$$(0.60 \times 0.10 + 0.40 \times 0.25)/(0.60 \times 0.05 + 0.40 \times 0.22) = 1.36,$$

whereas the SMR for area B is

$$(0.80 \times 0.10 + 0.20 \times 0.25)/(0.80 \times 0.05 + 0.20 \times 0.22) = 1.55,$$

whence area B would get more than A for any reasonable quantification of the SMR-based ranking of areas.

10. The Scottish report from Oxford<sup>2</sup> presents this as a damning counter-example to SMR—which it might well be, if the objective were a ‘geographical *description* of inequality’ in which the age-band populations were somehow irrelevant. But Table 1 uses the hypothetical data of the example to exonerate SMR, when the objective is switched to *allocate* funds to fight ‘inequality’ represented by big differences in age-band *poor-health indices* (ratios of age-band death-rates to the corresponding national death rates). For the economics of allocation, the size of targeted populations matters.

**Table 1: Poor-health indices for the Scottish example**

Area	Young age-band	Old age-band
A (60% young)	10/5 = 2.0	25/22 = 1.14
B (80% young)	9/5 = 1.8	24/22 = 1.09

If the cost of fighting ‘inequality’ in a particular area were roughly proportional to the number of individuals in the area who can be thought to need such intervention, the fact that area B (with the higher SMR) has many more such individuals than area A may be thought to justify higher *per capita* funding for area B (to meet the extra cost for the extra 20% in the young age-band).

11. For the analogous concerns of NHS England, the concept of an age-band’s poor-health index can throw light on the comparison of the SMR<75 of two MSOAs (A and B) in two special cases—but only when the arithmetic gives way to symbols. For a generic MSOA, there are four official statistics for each age-band—( $p_a$ ,  $P_a$ ,  $r_a$ ,  $R_a$ ), where  $p_a$ ,  $P_a$  are the respective proportions of the MSOA and national populations in age-band  $a$ , and  $r_a$ ,  $R_a$  are the corresponding death-rates. SMR<75

is the ratio of the *Sum of*  $p_a \times r_a$  in the numerator to the *Sum of*  $p_a \times R_a$  in the denominator, where the sums of products are taken over all the age-bands. The ratio can also be expressed as a single (weighted) sum of the MSOA's poor-health indices  $ph_a$ —namely the *Sum of*  $w_a \times ph_a$  where the weights  $w_a$  (necessarily adding to 1) are proportional to the products of  $p_a$  and  $R_a$ . Note that this alternative expression shows that  $SMR < 75$  gives the national death-rates  $\{R_a\}$  a weight equal to that of the MSOA age-band population proportions  $\{p_a\}$ .

12. The first special case is when A and B have the same poor-health indices  $\{ph_a\}$  and the difference  $SMR < 75_A - SMR < 75_B$  is then the *Sum of*  $(w_{aA} - w_{aB}) \times ph_a$ . This will be positive if A gives more weight than B to the (shared) unhealthier age-bands. It is tempting to conclude that this is clear confirmation of the DH/ACRA thesis that the larger SMR should get a larger allocation. But the weights  $\{w_a\}$  are mediated by  $\{R_a\}$ , which could exercise an influence on the order of *per capita* allocations suggested by the sign of  $SMR < 75_A - SMR < 75_B$ , reversing the order you would get if the weights were just the age-band population proportions.

13. In the second case, when A and B have the same age-profile  $\{p_a\}$  and therefore the same weight-profile  $\{w_a\}$ ,  $SMR < 75_A - SMR < 75_B$  is the sum of  $w_a(ph_{aA} - ph_{aB})$  over age-bands. In this case, the difference is in favour of A if, in their common weight-profile, the two MSOAs give enough weight to age-bands which have poorer health for A than for B. As in the first case, the same devil can tempt us here, and it requires the same caveat not to yield to temptation without further investigation.

14. All these considerations were accessible to the DH team and its ACRA advisors when they devised the public health formula that NHS England has now adopted, and they are still accessible to NHS England's finance officers. The 'elephant in the room' is the big question of why it is thought that a single index,  $SMR < 75$  (encasing MSOA age-band population proportions in a questionably rigid fashion) should be the prescriptive basis for allocating funds for such a multi-dimensional objective as fighting 'inequalities' in public health—which has a total allocation of about £6 billion in 2014-15. The essential stages in writing the prescription appear to have proceeded without any statement of rationale and can be outlined as follows.

15. My light-hearted but truthful synopsis is extracted from seven reports<sup>5,....,11</sup> and can be related for ease of exposition to Figure 1—a very pretty picture from reference 10:

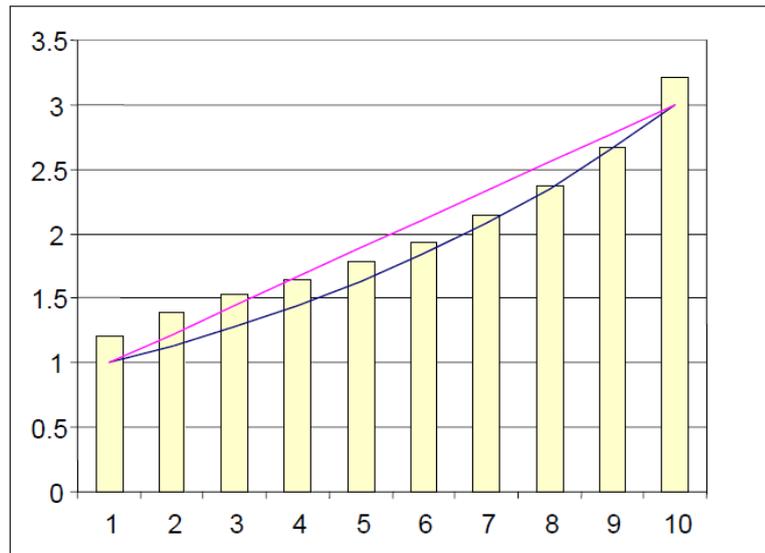


Figure 1: Interim illustration for the construction of SMR-based weights

The 7,000 or so MSOA SMR<75 values were ranked in increasing order and herded into 10 intervals containing, respectively, 5%, 14%, 19%, 16%, 12%, 10%, 8%, 6%, 5%, 5% of the MSOAs. (The smaller percentages towards the tails of the distribution of SMR<75 were a subjective accommodation of the spread in the tails that occasioned much argument between DH and its partners. The other percentages were those for equal-width intervals.) The yellow columns represent the 10 SMR<75 averages but the columns really extend to a base-line below the x-axis (deceiving as least one party to the partnership arguments). The pink line appears to have been fitted to the averages by the statistical method of least-squares. The portrayed interim labelling of the y-axis gave figures that increased about twice as steeply as the SMR<75 averages (forcing their true origin onto a base-line below the x-axis). NHS England has increased the 'twice' gearing factor from 2 to 4 (by changing the 3 on the y-axis to a 5), and uses the curved blue line (an exponential curve through the points (1,1) and (10,5)) to fix the MSOA weights for their allocations, taken to be proportional to the product of weight and population. The CCG allocations are then taken to be the sum of the constituent MSOA allocations.

This synopsis may be enough to make clear how the £6 billion is being spent. If it were also clear enough to dispose of the question of rationale, it would save us all the chore of searching for that holy grail through the many pages of those seven reports.

17. In conclusion, what are we to make of how NHS England is allocating 10% of over £60 billions going to CCGs? What the Finance Department has already done with the other 90% provides more than a clue. The disposal of the 90% has been shown<sup>4</sup> to be dominated by an irrational and evidence-free formula (PBRA3), whose financial implications are not even understood by the health economists who fabricated it. The prescription for the allocation of the remaining 10% comes from the Finance Department via ACRA, where health economists are well-represented. If you have the time, please speed-read references 5 to 11 and help me to locate the elusive rationale and the evidence that the new public health formula is more than an elaborate but poorly understood procedural deception. Between the lines, this note is suggesting that replacing SMR by CMF would be an irrelevant manipulation of the procedure, whose recorded history goes back to a ministerial intervention that allocated 15% of funds for 'unmet need' on the basis of *per capita* DFLEs (disability-free life expectancies) of the undivided populations of primary care trusts. May we hope that policy-makers will be able to break away from such sterile and politically contentious index-mongering—in favour of direct measurement of the states of health of stratified random samples of GP-registered individuals in the 200 or so CCGs? The Department for Work and Pensions has its quarterly Labour Force Survey that determines important and up-to-date national statistics with adequate precision and detail, by interviewing random samples of households over five successive quarters. An analogous survey of GP-registered individuals for NHS England should not need five interviews, and would be able to cover poor-health and health-care need at the same time, with far less cost than is now incurred by the wasteful misallocation of foolish formulas.

## References

<sup>1</sup> Sutton M, H. Gravelle, S. Morris, A. Leyland, F. Windmeijer, C. Dibben and M. Muirhead (2002), *Allocation of Resources to English Areas: Individual and small area determinants of morbidity and use of healthcare resources*. Edinburgh: Information and Statistics Division of NHS Scotland.

<sup>2</sup> *Scottish Indices of Deprivation 2003*, Scottish Government

<http://www.scotland.gov.uk/Publications/2003/02/16377/18194> (retrieved March 2014)

<sup>3</sup> For example, see Galbraith, G. and Stone, M. (2011), *The abuse of regression in the National Health Service allocation formulae: response to the Department of Health's 2007 'resource allocation research paper'*, in *Statistics in Society*, Royal Statistical Society <http://onlinelibrary.wiley.com/doi/10.1111/j.1467-985X.2010.00700.x/pdf> (retrieved March 2014);

Stone, M. (2010), *Formulas at war over two sorts of inequality in health funding*, Civitas.

<http://www.civitas.org.uk/pdf/formulasatwarApril2010.pdf> ;

Stone, M. (2010), *Failing to Figure*, Civitas.

<sup>4</sup> Stone, M. (2013), *Explicating 'wrong' or questionable signs in England's NHS funding formulae: correcting wrong explanations*, Civitas

<http://www.civitas.org.uk/nhs/download/wrongsigns.pdf>

<sup>5</sup> *Allocation of resources to NHS England and the commissioning of services for 2014/15 and 2015/16*, NHS England Board Paper NHSE 121305, December 2013.

<http://www.england.nhs.uk/wp-content/uploads/2013/12/bm-item7.pdf> (retrieved March 2014)

<sup>6</sup> *Exposition Book Public Health Allocations 2013-14 and 2014-15: Technical Guide*, Department of Health, January 2013

[https://www.gov.uk/government/uploads/system/uploads/attachment\\_data/file/213324/Public-Health-Weighted-Capitation-FormulaTechnical-Guide-v0.13.pdf](https://www.gov.uk/government/uploads/system/uploads/attachment_data/file/213324/Public-Health-Weighted-Capitation-FormulaTechnical-Guide-v0.13.pdf) (retrieved March 2014)

<sup>7</sup> *Exposition Book Public Health Allocations 2014-15*, Department of Health Excel file, April 2013.

[https://www.gov.uk/government/uploads/system/uploads/attachment\\_data/file/190643/Exposition\\_Book\\_Public\\_Health\\_Allocations\\_2014-15\\_April\\_2013.xlsx](https://www.gov.uk/government/uploads/system/uploads/attachment_data/file/190643/Exposition_Book_Public_Health_Allocations_2014-15_April_2013.xlsx) (retrieved March 2014)

<sup>8</sup> *Healthy Lives, Healthy People: Update on Public Health Funding: Public health exposition book based on ACRA's interim recommendations: technical guide*, Department of Health, June 2012 <https://www.gov.uk/government/publications/healthy-lives-healthy-people-update-on-public-health-funding> (retrieved March 2014)

<sup>9</sup> *Engagement on Interim Public Health Recommendations*, Department of Health, June 2012

[https://www.gov.uk/government/uploads/system/uploads/attachment\\_data/file/213328/ACRA201214-Engagement-on-interim-public-health-recommendations-na.pdf](https://www.gov.uk/government/uploads/system/uploads/attachment_data/file/213328/ACRA201214-Engagement-on-interim-public-health-recommendations-na.pdf) (retrieved March 2014).

<sup>10</sup> *Public Health—Finalisation of Formula: Introduction and Action for ACRA*, Department of Health, August 2012

[https://www.gov.uk/government/uploads/system/uploads/attachment\\_data/file/213331/ACRA\\_201216-Public-Health-Finalisation-of-formula.pdf](https://www.gov.uk/government/uploads/system/uploads/attachment_data/file/213331/ACRA_201216-Public-Health-Finalisation-of-formula.pdf) (retrieved March 2014).

<sup>11</sup> *Public Health Formula—SMR Weights: Introduction and Action for ACRA*, Department of Health, September 2012

[https://www.gov.uk/government/uploads/system/uploads/attachment\\_data/file/213334/ACRA\\_201219-PH-formula-SMR-weights.pdf](https://www.gov.uk/government/uploads/system/uploads/attachment_data/file/213334/ACRA_201219-PH-formula-SMR-weights.pdf) (retrieved March 2014).

<sup>12</sup> Fox, A. J. (1978), *Occupational Mortality*, London: HMSO